

## 11-5 Square Roots of Variable Expressions

**Objective:** To find square roots of variable expressions and to use them to solve equations and problems.

### Property

**Property of Square Roots of Equal Numbers** For any real numbers  $r$  and  $s$ :  
 $r^2 = s^2$  if and only if  $r = s$  or  $r = -s$ .

**CAUTION** When you are finding the principal square root of a variable expression, you must be careful to use absolute value signs when needed to ensure that your answer is positive. For example,  $\sqrt{x^2} = |x|$ , not  $x$ .

**Example 1** Simplify: a.  $\sqrt{144x^2}$  b.  $\sqrt{25n^8}$  c.  $\sqrt{12a^3}$

**Solution** a.  $\sqrt{144x^2} = \sqrt{144} \cdot \sqrt{x^2}$   
 $= 12|x|$

b.  $\sqrt{25n^8} = \sqrt{25} \cdot \sqrt{n^8}$   
 $= \sqrt{25} \cdot \sqrt{(n^4)^2}$   
 $= 5n^4$  ( $n^4$  is always nonnegative)

c.  $\sqrt{12a^3} = \sqrt{4 \cdot 3 \cdot a^2 \cdot a}$   
 $= \sqrt{4} \cdot \sqrt{a^2} \cdot \sqrt{3} \cdot \sqrt{a}$   
 $= 2|a|\sqrt{3a}$

### Simplify.

1.  $\sqrt{81x^2}$

2.  $\sqrt{121x^2}$

3.  $\sqrt{20x^2}$

4.  $\sqrt{45x^4}$

5.  $-\sqrt{25x^2}$

6.  $-\sqrt{16c^4}$

7.  $-\sqrt{64d^8}$

8.  $-\sqrt{98n^6}$

9.  $\sqrt{225y^4}$

10.  $\sqrt{400a^6b^4}$

11.  $\sqrt{81m^{12}}$

12.  $\sqrt{441n^6}$

13.  $\pm\sqrt{75x^2y^3}$

14.  $\pm\sqrt{60x^6y^4}$

15.  $-\sqrt{121x^2y^2}$

16.  $-\sqrt{900a^4b^6}$

17.  $\pm\sqrt{\frac{81x^8}{100}}$

18.  $\pm\sqrt{\frac{121}{225x^{10}}}$

19.  $\sqrt{\frac{x^4y^8}{9z^2}}$

20.  $\sqrt{\frac{32m^3n^2}{2mn^2}}$

21.  $\sqrt{\frac{16x^{18}}{3600y^{20}}}$

22.  $\sqrt{\frac{256x^{40}}{25}}$

23.  $\sqrt{2.25x^4}$

24.  $-\sqrt{2.56k^2}$

### 11-5 Square Roots of Variable Expressions (continued)

**Example 2** Simplify  $\sqrt{m^2 - 8m + 16}$ .

**Solution**  $\sqrt{m^2 - 8m + 16} = \sqrt{(m - 4)^2} = |m - 4|$

Simplify.

25.  $\sqrt{x^2 + 4x + 4}$

26.  $\sqrt{n^2 - 14n + 49}$

27.  $\sqrt{x^2 - 6x + 9}$

28.  $\sqrt{m^2 - 10m + 25}$

**Example 3** Solve  $4x^2 = 25$ .

**Solution 1**

$$\begin{aligned} 4x^2 &= 25 \\ 4x^2 - 25 &= 0 \\ (2x + 5)(2x - 5) &= 0 \\ 2x &= -5 \quad \text{or} \quad 2x = 5 \\ x &= -\frac{5}{2} \quad \text{or} \quad x = \frac{5}{2} \end{aligned}$$

Check:  $4\left(\frac{5}{2}\right)^2 \stackrel{?}{=} 25$       and       $4\left(-\frac{5}{2}\right)^2 \stackrel{?}{=} 25$

$25 = 25 \checkmark$       and       $25 = 25 \checkmark$

The solution set is  $\left\{\frac{5}{2}, -\frac{5}{2}\right\}$ .

**Solution 2**

$$\begin{aligned} 4x^2 &= 25 \\ x^2 &= \frac{25}{4} \\ x &= \pm\sqrt{\frac{25}{4}} \\ x &= \pm\frac{5}{2} \end{aligned}$$

Solve.

29.  $x^2 = 16$

30.  $n^2 = 36$

31.  $x^2 - 9 = 0$

32.  $d^2 - 25 = 0$

33.  $0 = a^2 - 49$

34.  $0 = m^2 - 64$

35.  $2m^2 - 18 = 0$

36.  $40b^2 - 160 = 0$

37.  $36y^2 - 16 = 0$

38.  $4c^2 - 25 = 0$

39.  $0 = 49z^2 - 9$

40.  $0 = 45x^2 - 125$

### Mixed Review Exercises

Simplify.

1.  $\pm\sqrt{80}$

2.  $-4\sqrt{75}$

3.  $3\sqrt{256}$

4.  $2^{-3} - 3^{-2}$

5.  $4^3 \cdot 2^{-5}$

6.  $(3x^2)^3(-2x^4)^2$

Evaluate if  $x = 9$ ,  $y = 16$ , and  $n = 1$ .

7.  $x^2 + y^2$

8.  $x^2n^2$

9.  $y^2 - x^2$

10.  $\sqrt{\frac{y}{n}}$

11.  $\sqrt{\frac{x}{y}}$

12.  $(\sqrt{y})^2$